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THE MATHEMATICS TEACHER

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AN IMPORTANT CONTRIBUTION TO THE TEACHING OF MATHEMATICS

The National Council of Teachers of Mathematics has made an important contribution to the teaching of elementary mathematics through the publication of its *First Year Book.* The general theme of the book is a General Survey of Progress in the Last Twenty-Five Years. Professors David Eugene Smith, Eliakim Hastings Moore, Raleigh Schorling, William David Reeve, Frank Clapp, Herbert E. Slaught, Miss Marie Gugle, Mr. William Betz and Mr. Edwin W. Schreiber are the contributors.

Professor Smith discusses the early attempts at improving the syllabi, the influences for the betterment of the courses, the work of the National Committee, the work of the International Commission, the influence of the College Entrance Board and progress in arithmetic, algebra and geometry.

In December, 1902, Professor Eliakim Hastings Moore delivered a presidential address before the American Mathematical Society entitled *On the Foundations of Mathematics.* The influence of this address upon secondary school mathematics has been so far-reaching and is yet so pertinent to clear thinking about the "object" and the "subject" of mathematics in secondary schools that it has been reprinted in order to satisfy the obvious need to have it available for teachers generally.

Professor Schorling presents the results of one of the best objective studies in curriculum making that has yet been made. He shows how the detailed materials of a course of study in junior high school mathematics have been determined.

One of the important aspects of progress during the last twenty-five years has been the development of methods of measuring the quality of instruction. Professor Reeve shows how tests in mathematics may be made more effective by pointing out the advantages and disadvantages of difficult types of tests.

Nowhere in the school curriculum has progress been more immediate and far-reaching than in the junior high school grades. William Betz of Rochester, New York, a city which has made

noteworthy achievement in the development of junior high schools, discusses the whole problem of mathematics in the junior high schools. Among other topics he considers the struggle between stratified and unified work in mathematics, project work and intuitive geometry.

Professor Clapp summarizes and interprets some of the recent investigations that bear upon the teaching of arithmetic. This section will be of interest to teachers of both secondary and elementary schools.

The importance of mathematics is not adequately appreciated by the general public. To be sure, the strategic importance of mathematics became recognized during the World War. The great extent to which the arts and sciences of peace are dependent upon mathematics is pointed out by Professor Slaught in the article entitled, *Mathematics and the Public.* High school teachers will wish to give publicity to the point of view as well as the facts set forth by Professor Slaught.

The increased use of mathematical recreations as a means for developing an interest and appreciation for mathematics is evidenced by the large number of articles that have been written on mathematics clubs in junior and senior high schools during the recent years. Assistant Superintendent Marie Gugle and others of Columbus, Ohio, contribute a section entitled, *Recreational Values Achieved Through Mathematics Clubs in Secondary and Elementary Schools.*

A very complete bibliography of mathematics books published in recent years for pupils and teachers is contributed by Mr. Edwin W. Schreiber, of Maywood, Illinois.

Much credit is due the Year Book Committee consisting of Charles M. Austin, Harry English, William Betz, Walter C. Eells and Frank C. Touton and to Professor Schorling, President of the National Council during the preparation of the Year Book, for this major contribution to our professional literature. This *Year Book* along with the report of the National Committee of Mathematical Requirements, should be read and reread by every teacher of mathematics in junior and senior high schools. The two publications will doubtless form a part of the required reading in professional courses. J. R. C.

Copies of the Year Book may be secured from Mr. Charles M. Austin, Chairman, High School, Oak Park, Illinois, at $1.10 per copy.

HERESY AND ORTHODOXY IN GEOMETRY

GEORGE W. EVANS
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In discussing the content of school geometry, we may well take as a basis what has always been the basis, namely, the systematic treatise on geometry that was written by Euclid about 300 B. C. America and France have been in the habit of following Legendre's arrangement of this; England has until the present century followed Euclid faithfully; but everywhere more than half of the ancient text has been omitted.

There is perennial dispute about Euclid. People like Augustus De Morgan and our own David Eugene Smith regard Euclid's text-book as the best foundation for our work in mathematics; others look upon it as a stone of stumbling and a rock of offence. Let us make clear to ourselves what Euclid really did; and then decide, if we can follow him, how far it is best to do so.

First, he was a compiler; but what a compiler! He found geometry a scattered possession; arithmetic partly of the utilitarian sort, partly speculative and thus commendably abstract and useless; a theory of proportion, substantially complete, from the hand of Eudoxus; an extensive theory of quadratic surds, the work of Theaetetus. There were other mathematical topics that he could have used, but he took only these four as material for his book. He selected his postulates with extraordinary insight, and developed the whole in one logical system, not without some valuable contributions of his own. Thus he made his textbook, the first to view arithmetic, algebra and geometry as one subject introductory to mathemathical learning. Advocates of "fusion," lay this to your heart!

We may as well admit that this book was written for mature philosophers originally, however soon it was afterwards used by boys of fourteen. But in those days it took a mature philosopher to multiply 25 by 25, to solve such equations as $2x + 1 = 17$, to understand a thing like $2\frac{3}{4}$, or to realize that $\sqrt{2}$ was anything at all. All this was due to the fact that their arithmetic was clumsy, their algebra was almost all talk, and their units of measure were geographical accidents. Yes, even to follow, in

Euclid's lifetime, any considerable part of his teaching, one had to be a mature man, a lover of wisdom, and a persistent thinker.

In modern times a boy of fourteen has units of measure that everybody knows about, and he has a symbolism for arithmetic and a symbolism for algebra that make easy for him many things that were extremely difficult for the adult neophyte in Alexandria. He has more and better apparatus for his thinking in mathematics than the ablest philosopher had when Alexandria was the university town of the ancient world.

Even at that there are one or two things in the old book that have only recently been thoroughly understood. It took nearly all those twenty-two centuries to convince the world that his axiom of parallels had to be an axiom, or that his definition of proportion actually implied the existence, and furnished the basis for a complete understanding, of what we now call real number. We shall have to admit that boys of fourteen will require good teaching for a long time before they can get those two ideas clearly in mind. However, for most of the material in Euclid, our modern advantages give us command of a much more lucid and coherent treatment than his; it is by all means advisable to use it.

In order to see what Euclid was trying to do, we must consider, not only "the first six books with portions of the eleventh and twelfth," as the title-pages in England used to say it, but the whole thirteen. We shall see a great many familiar theorems there, but the number we shall not find will surprise us. Probably more than half of the theorems in the average American school book will not be found in Euclid. He left out the whole subject of spherical triangles, the whole subject of loci, and the whole subject of mensuration, as well as a good many other theorems here and there that we now think worth while. In many cases he left these things out because he did not know about them; but he certainly knew something about mensuration.

There are 465 theorems printed in Heiberg's edition of Euclid's text: less than a quarter of them are studied now; evidently they dealt with something besides what we call elementary geometry. We begin to see the difference in Book II. The first ten theorems here set up the rules of an algebra such as Euclid

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Read at the February, 1926, meeting of the National Council of Teachers of Mathematics, Washington, D. C.

thereafter generally uses. Multiplication is the construction of a rectangle of which the sides are the factors; division is the construction of a rectangle equal to the dividend on the divisor as a base; square root is the side of a square constructed equal to a given rectangle, and so on. Such is the effect, though not the stated purpose, of these ten theorems.

Again, in Book V, he starts with the theory of proportion that Eudoxus invented to deal with that scandalous hypotenuse. He applies it to similarity theorems, getting proofs that would look weird to our graduates. He deals extensively with integers, even and odd, prime and not prime, with rational fractions and with irrationals. Sometimes his lines represent numbers explicitly, just as letters do in our algebra, and geometry does not appear in the argument; again the lines are just lines, parts of a geometric figure, and the inferences are drawn by geometry; but what he is obviously doing is with great care setting up a systematic exposition of the correspondence between number and geometric magnitude. In this way he clearly shows an enlargement of the idea of number, though the new sorts of number were to him only ratios or relative lengths. Thus one of his theorems about irrational numbers would state that if a certain line is rational, a certain other line is the irrational line known by the name so-and-so.

By Euclid's definition numbers were integers. A fraction with numerator 1 he called a part, and a proper fraction with numerator more than one he called "parts." An incommensurable number he could speak of only as a line or as the ratio of two lines. He would be likely to follow the fashion of Plato's disciples, among whom were his Eudoxus and Theaetetus, in saying that the square root of 50 was something more than 7, but that it actually was expressible only as a line. Because he had practically no algebraic symbols, he had names for different kinds of irrational number; for example, the sum of a square root and an integer, like $\sqrt{2} + 3$, was a "line of two terms," or, in our wording, a binomial; the number obtained by subtracting an integer from a square root was an apotome (a "cut-off"). There were more than twenty different kinds of radical expressions thus enumerated, each name distinguishing not a particular number, but a particular class of algebraic expressions.

These classes are somewhat more comprehensive than I have described. An expression like $\sqrt{2} + \sqrt{3}$ would be for Euclid the same kind of thing as $\sqrt{2} + 3$; for each of them is the sum of two numbers whose squares are commensurable, but not the numbers themselves. Whether one of the numbers was itself commensurable with the unit, he did not take the trouble to inquire. One reason for this is, that except in the theorems referring to prime numbers and multiples, Euclid used no unit.

The five regular solids,—the so-called Platonic bodies,—had in Plato's time an importance out of all proportion to the attention we give them now. They formed a closed field of perfect types, standing as symbols of completed thought. Theaetetus, who was of Plato's school, had analyzed the algebraic relations of certain lines in these figures, and Euclid made them the last topic in the last book of the Elements. Here we find the lengths of these lines, relative to the diameter of the circumscribing sphere, identified with known types of irrationals; and there is even a diagram in which each of them is laid off to compare with the diameter,—a surprising anticipation of our familiar "graphical method." The accompanying discussion contains a sort of tabulation of their mutual ratios. The subject seems to form the goal and apex of the whole treatise, and the early commentators did so consider it; we are more interested, however, to see in it a conspicuous opportunity, which Euclid seized, to display the method of dealing with irrationals. It is the spectacular finish of his pursuit of number.

If anybody will take a week or so, and starting with Euclid's definitions of these different kinds of numbers, follow the arguments through, he will have no doubt of the maturity or the acumen of the men who studied them in ancient times. On the other hand, if he uses the ordinary notation of elementary algebra, he can get the same results by comparatively simple means. Of all these results many are now of little importance, and some are trivialities; but if they were all of interest, it would not be reasonable to expect our pupils to obtain them after Euclid's manner.

In our modern text-books also, as in Euclid, there seems to be a goal or climax towards which the succession of theorems proceeds. It is the measurement of the circle in plane geometry,

and of the sphere in solid geometry. Now Euclid himself had a theorem that circles were proportional to the squares on their radii; but he did not apply to this his proportion "by alternation," so as to show that the ratio of a circle to the square on its radius is always the same, perhaps because he was in doubt about the circle and the square being the same kind of magnitude; consequently, although he probably had such a number as π in mind, he did not see a way to fit it into his logical system and he made no mention of it.

I think, however, if he were here, he would approve the search for this number. Let us hope so. We are going after it anyway. He would certainly not care for the practical uses we might find for the value of π; but I think he would be very much interested in the fact that this is an entirely new kind of irrational number, not one of his famous incommensurables or anything like them, but still a number that can be obtained by the use of that immortal definition of proportion.

Another thing about the character of Euclid's text-book, a thing that does not need to be searched for, is the fact that the subject hangs together. Upon the axioms and definitions depend some of the propositions, others upon those, and others again; so that there is in the book, leading up to some important theorem, a chain of successive theorems each of which is proved by one or more preceding. This is the thing our geometry can teach best, the lesson of persistent deduction. It is a very striking example of scientific procedure, the very thing our pupils should have.

On the other hand, this succession is not continuous in Euclid. Sometimes several theorems succeed each other in this way, at other times we come to a dead end and start off from a new origin. There are whole regions that seem largely cut off from what precedes or follows; and there are some theorems in the later books that go back to first principles. It is advisable for us to make the idea of successive inference more obvious; it can certainly be done without sacrificing any other pedagogic advantage.

Euclid did not use any unit in the strictly geometrical parts of his book, not even when he was dealing with irrational expressions. Now Plato makes Theaetetus say, in talking things over with Socrates, that "squares containing three square feet

or five square feet are not commensurable in length with the unit of the foot." Here is one of Euclid's sources, and here is the standard unit of measure for length used in abstract discussion. Yet Euclid did not so use it. One reason why he did not may be found in the fact that Theaetetus was an Athenian in Athens, where the word foot had a definite meaning; while Euclid was quite possibly not an Athenian, and his teaching was done in Alexandria, to students that came from everywhere in the fragrant and fruitful East. Their units of measurement would vary with their origin; they might have used the cubit, the span, the digit, or something else, and the units of the same name might not mean the same thing. Even in modern times there is something queer about such units as the ounce, the bushel, and the ton, though our measures of length are unequivocal. But there was no standard unit known to all or to most of the students in Alexandria.

Another reason may have been Euclid's aversion to practical uses of his science, which was apparently greater even than Plato's; and Plato repeatedly sneers at those who find in practical applications the main purpose of scientific thought. That point of view was characteristic of the Greeks, but we have somewhat outgrown it. Even if we had not, the assumption of one line as the unit of length, and of one angle as the unit of angle, would not injure the scientific value of our subject; and there is no reason why such a unit should not have the name by which it is known to all learned men, nor any reason why we should not use one of the well-known units now, and another again. If we include among the aims of our teaching, as Euclid did, the elucidation of number by means of geometry, the use of such units will materially simplify our work without in the slightest degree detracting from its scientific character.

Over and over again Euclid is criticized because he does not show how he came at his proofs, or teach his pupils how to invent them. This appears to be true, if we look only at the parts of the Elements usually studied. There is, however, in some of the interpolations made in Book XIII, a definite indication that "analysis" was used in expounding the subject, for instruction was given by oral discussion only; and one of Euclid's other writings, the "Data," which is still obtainable, is especially designed as a guide to the discovery of proofs in study following

the Elements. At any rate, if this is an acknowledged fault in the Elements, we are under no obligation to be guilty of it ourselves.

We can follow Euclid, then, in what seems to be his main purpose, the correlation of number with geometric magnitude, for developing and clarifying the ideas of fractional and incommensurable number; we can follow him in the careful structure of a deductive system, with a clearer view than he apparently had into some of the dark corners; we can use ratios of straight lines, and give those ratios names, like sine and cosine, which are not the names of particular ratios, but of classes of ratios. We can, in our modern way, accept the simplification that comes by making one term of the ratio a unit, and finally we can consider the ratio a definite and single number instead of a pair of numbers; we can add to the continent of the subject, as it was when he explored it, a few peninsulas and islands, discovered since; and when it comes to the two famous difficulties, the theory of parallels and the definition of proportion, we may if we choose postpone the thorough survey of those more solitary regions until our followers have a wider range of vision.

To make these general statements definite, the following program of innovation is proposed:

(1) Treat the subject as a mensuration raisonnée, beginning with the measurement of a straight line as the first step in the geometric study of number. (2) Do not hesitate to use trigonometric functions as orthodox geometric ratios. (3) Make the determination of π the culmination of the year, and make the succession of theorems that are necessary for that the obvious high road, other theorems being reached by enticing side trips.

THE INDUCTIVE METHOD IN JUNIOR COLLEGE MATHEMATICS

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To prove the general case first and then to show the application to specific problems is the usual technique of development of a Mathematical idea. This deductive presentation of mathematical ideas is so concise, so beautiful and so logical that nearly all textbook writers and most teachers enjoy using it. In spite of its universality of use the deductive method of thinking is not the way in which ideas were first discovered, nor is it the way in which they are most easily learned.

The writer just concluded an experiment with presenting the factor theorem in Advanced Algebra deductively and inductively. It was then ascertained which development was the simplest to understand and which was the most convincing. Almost unanimously they voted in favor of the inductive development. It shall not be the object of this paper to report on that whole experiment, but to report merely the inductive part of it.

A simple expression was written on the board and the class instructed to watch carefully to see how quickly they could get a new idea for factoring. The following division was made,—

$$\begin{array}{l} x - 5)x^2 - 2x - 24(x + 3 \\ \qquad\;\; \underline{x^2 - 5x \quad} \\ \qquad\qquad\; + 3x - 24 \\ \qquad\qquad\quad\; \underline{3x - 15} \\ \qquad\qquad\qquad\; - 9 \text{ remainder} \end{array}$$

Suppose now we substitute $x = + 5$ in the original expression $5^2{}_2 - 2.5 - 24 = 9$. How does this result compare with the remainder in the division? Try other divisors.

$$\begin{array}{l}
x - 10)\,x^2 - 2x - 24\,(x + 8 \\
x^2 - 10x \\
\hline
\phantom{x - 10)\,x^2 - {}}8x - 24 \\
\phantom{x - 10)\,x^2 - {}}8x - 80 \\
\hline
+ 56 \text{ remainder}
\end{array}$$

If $x = + 10$, then $x^2 - 2x - 24 = 10^2 - 2.10 - 24 = + 56$.

$$\begin{array}{l}
x + 7)\,x^2_2 - 2x - 24\,(x - 9 \\
x^2 + 7x \\
\hline
- 9x - 24 \\
- 9x - 63 \\
\hline
+ 39
\end{array}$$

If $x = - 7$, then $x^2 - 2x - 24 = (- 7)^2 - 2\,(- 7) - 24 = 49 + 14 - 24 = 39$.

By this time the class will be getting the idea that if you divide a function of x by x minus some number, the remainder will be the same as the result obtained by substituting that number for x in the original expression. Notice that the sign of the number is changed in the substitution.

$$\begin{array}{l}
x - 6)\,x^2 - 2x - 24\,(x + 4 \\
x^2 - 6x \\
\hline
+ 4x - 24 \\
\phantom{x - 6)\,x^2 + {}}4x - 24 \\
\hline
0
\end{array}$$

If $x = + 6$ then $x^2 - 2x - 24 = 6^2 - 2.6 - 24 = 0$.

What does this suggest about factors? Try to get the class to discover the factor theorem by themselves from this presentation. It will be an extremely dull class or a presentation lacking in enthusiasm, if someone does not get the idea that you can work this idea backwards to find factors.

Present now a cubic expression.

$$
\begin{array}{l}
x-1)\,x^3-3x^2-x+3\,(x^2-2x-3 \\
\underline{x^3-x^2} \\
-2x^2-x \\
\underline{-2x^2+2x} \\
-3x+3 \\
\underline{-3x+3} \\
0
\end{array}
$$

"Clearly $x - 1$ is a factor because the remainder is zero. How else could we have shown what the remainder would be?" If $x = +1$ then $x^3 - 3x^2 - x + 3 = 1 - 3 - 1 + 3 = 0$. Let us try some other substitutions to find the other binomial factors if there are any more.

If $x = -1$ then $x^3 - 3x^2 - x + 3 = (-1)^3 - 3(-1)^2 - (-1) + 3 = -1 - 3 + 1 + 3 = 0$, and therefore $x + 1$ is another factor because the remainder is zero when the substitution $x = -1$ is made.

If $x = -3$ then $x^3 - 3x^2 - x + 3 = (-3)^3 - 3(-3)^2 - (-3) + 3 = -27 - 27 + 3 + 3 = -48$ and therefore $x + 3$ is not a factor because the remainder is not zero. If $x = +3$ then $x^3 - 3x^2 - x + 3 = 3^3 - 3.3^2 - 3 + 3 = 27 - 27 - 3 + 3 = 0$ and therefore $x - 3$ is another factor because the remainder is zero. The factors are then $(x - 1)(x + 1)(x - 3)$.

Why would it be useless to try $(x - 2)$ or $(x + 5)$ as factors? Clearly then the numbers which we substitute for x to find the factors of the expression in x must themselves be factors of the constant term.

Factor $x^4 + 2x^3 - 7x^2 - 8x + 12$. Get suggestions from the class of possible factors. Since the divisors of $+12$ are ± 1, ± 2, ± 3, ± 4, ± 6, ± 12, and since x^4 suggests that there may be four factors, they must be four of these if integral: $x - 1$, $x + 1$, $x - 2$, $x + 2$, $x - 3$, $x + 3$, $x - 4$, $x \pm 4$, $x - 6$, $x + 6$, $x - 12$, $x + 12$. Why?

If $x - 1$ is a factor, then $+1$ substituted for x in the original expression will reduce it to zero.

If $x = +1$ then $1^4 + 2\,(1)^3 - 7\,(1)^2 - 8\,(1) + 12 = 0$, and $x - 1$ is a factor.

If $x = -1$ then $(-1)^4 + 2\,(-1)^3 - 7\,(-1)^2 - 8\,(-1) + 12 = 12$, and $x + 1$ is not a factor.

If $x = +2$ then $2^4 + 2\,(2)^3 - 7\,(2)^2 - 8\,(2) + 12 = 0$ and $x - 2$ is a factor.

If $x = -2$ then $(-2)^4 + 2\,(-2)^3 - 7\,(-2)^2 - 8\,(-2) + 12 = 0$ and $x + 2$ is a factor.

If $(x - 1)\,(x - 2)\,(x + 2)$ are three factors, by inspection the fourth must be $(x + 3)$ since $(-1)\,(-2)\,(+2)\,(+3) = 12$, the last term. Try it. If $x = -3$ then $(-3)^4 + 2\,(-3)^3 - 7\,(-3)^2 - 8\,(-3) + 12 = 0$, and $x + 3$ is a factor.

The following problems were assigned to be worked at their seats in class. The purpose of this was to furnish drill on the work presented and to give the teacher a chance to give individual help to the slower ones. As soon as a few of the class had all the problems worked, the correct answers were read and each student scored his own work.

1. $x^2 - 3x + 10$.
2. $x^3 - 5x^2 + x - 5$.
3. $x^3 + 4x^2 + x - 6$.
4. $x^3 + 2x^2 + 3x + 2$.
5. $x^3 - 3x - 2$.

More theory was needed in the case of problem four, where there is only one binomial factor. This was given to the class as a whole, and then the advance assignment was made. That assignment consisted of twelve problems, two of which were quartics. The remainder of the period, about fifteen minutes, was then used for preparation of the advance assignment under the supervision of the teacher.

The general truth in this presentation was learned by its application to specific problems. The proof in the text was referred to as another way of explaining the same material. After this inductive presentation, a drill and a study period were provided for during the class hour to provide for individual differences and individual help, and also to give a chance to present any additional theory necessary. Such was the technique that produces the greatest interest and the best results.

HOW MUCH ELEMENTARY ALGEBRA IS REMEMBERED BY FRESHMEN WHEN ENTERING COLLEGE?

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The issue of THE MATHEMATICS TEACHER for March, 1925, contains an article by Gordon R. Mirick and Vera Sanford reporting the results for high school students of two short tests covering items of fundamental importance in first year algebra and in arithmetic. The object of this paper is to report the results of giving the first of these two tests to a group of freshmen in Whitman College, and to compare these results with those reported for the high school students in the article mentioned.

The test was given in the first week of college, in the fall of 1925, to every member of the freshman class in mathematics. This group comprised 85 students out of a freshman class of 221. Thus, only one-third of the class elected mathematics. Forty minutes was allowed for the test, which was ample time for all but a few of the very slowest to finish it easily.

Of the group tested, 21 had had only a single year of elementary algebra, while the remaining 64 had had one and one-half years or more of it. Seven reported two years or more. Results are reported for these two groups separately.

The 35 questions of the test are given below, arranged in the same 12 groups as in the original paper by Mr. Mirick and Miss Sanford. Accompanying them are given the percentage of correct responses for the two groups of college students, and the median percentage of correct responses for the high school group. Of the high school group, 41% had had one year of algebra, 14% one and one-half years, 28% two years and 17% two years of correlated mathematics when the test was given.

PART I. LAWS OF EXPONENTS

Underline the correct answer

1. $x^2x^3 = x^6, x^5, 2x^2$.
2. $r^{12} \div r^4 = r^3, r^8, r^{16}$.
3. $(4x^3)^2 = 8x^5, 16x^5, 8x^6$.

	1 Year Group	1½ Year Group	High School Group
1	71	89	74
2	33	80	55
3	67	91	83

The results were very satisfactory for the group of freshmen with one and one-half years of algebra; they were fair for the one-year group, except in the second problem. Only one-third of this group could select the correct quotient of three suggested ones for this simple problem in division.

PART II. FRACTIONS

Underscore the correct answer

4. $\frac{1}{a} + \frac{1}{b} = \frac{2}{ab}, \frac{a+b}{ab}, \frac{2}{a+b}$

5. $\frac{1}{a} \div \frac{1}{b} = \frac{b}{a}, \frac{1}{ab}, \frac{a}{b}$

6. $\frac{1}{a} - \frac{1}{b} = \frac{0}{a-b}, \frac{1}{a-b}, \frac{b-a}{ab}$

	1 Year Group	1½ Year Group	High School Group
4.	14	41	32
5.	57	81	63
6.	19	39	28

In the next examples, reduce your answers to the lowest terms.

7. $\frac{2}{3} - \frac{5}{9} =$
8. $\frac{7}{4} - \frac{3}{5} =$
9. $\frac{8}{11} \div \frac{7}{22} =$
10. $\frac{8}{3} \times \frac{21}{64} =$
11. $\frac{5}{3} + \frac{3}{2} - \frac{3}{4} =$

	1 Year Group	1½ Year Group	High School Group
7.	71	89	82
8.	62	84	76
9.	38	72	74
10.	57	84	70
11.	76	83	72

The inability of the two freshmen groups to handle these very simple types of algebraic fractions is absolutely disgraceful. It is the poorest showing of any of the twelve groups of questions. While four-fifths of the better group managed to select the correct answer to the division problem, only two-fifths of them could do it for the simple problem in addition (No. 4) or subtraction (No. 6). The showing of the one-year group is too bad for comment. Pure chance, where only three choices are possible, should give a much better result! For the combined groups, 55% marked the third answer to problem 4, while 22% thought the first answer to problem 6 was correct!

With reference to the second group, the purely arithmetical fractions, the comments of Mr. Mirick and Miss Sanford may well be repeated, with even greater point since the freshman group was only that one-third of the entire class who elected mathematics. They say:

"With the arithmetic fractions, the case appears better, but it really is worse. These pupils have been studying common fractions ever since they were in the fourth grade. If anything on this test ought to have been learned to high mastery, it is this subject which they have studied at intervals for a minimum of six years."

One wonders what is the ability with common fractions of the two-thirds of the freshman class who did not elect mathematics!

Part III. Decimals

12. *Multiply* 3.1416
0.041

13. *Divide* $0.006\overline{)\,62.832}$

	1 Year Group	1½ Year Group	High School Group
12.	67	83	70
13.	67	84	68

The situation with reference to decimal fractions is a little better, but not markedly so for either of the freshman groups.

Part IV. Negative Numbers

14. What is the sum of 5 and -9?
15. From -3 subtract -4.
16. What is the product of -7 and 2?
17. Divide -30 by -12.

	1 Year Group	1½ Year Group	High School Group
14.	86	95	86
15.	33	58	48
16.	62	86	70
17.	81	86	77

Both groups of freshmen remember the technique of dealing with simple negative numbers fairly well, except in the case of subtraction (No. 15). This also proved the stumbling block for the high school group.

Part V. Substitution

18. If a is 4 and b is 3, then $4ab =$?
19. If x is 6 and y is 4, then $2x - y =$?

	1 Year Group	1½ Year Group	High School Group
18.	81	100	86
19.	81	97	88

The results for this part of the test were the best in any of the twelve parts. They are highly satisfactory for the 1½-year group, and fairly so for the 1-year group.

Part VI. Equations

20. $2x = 14$, $x =$?
21. $2x + 3 = x + 7$, $x =$?
22. $3x - 4 = 0$, $x =$?
23. $6/x = 2$, $x =$?
24. $x/4 + x/3 = 7$, $x =$?

	1 Year Group	1½ Year Group	High School Group
20.	100	100	98
21.	81	97	86
22.	81	89	72
23.	86	95	80
24.	43	70	50

The showing on these simple equations in one unknown is very satisfactory, except for the last one. No. 20 is the only problem in the entire test solved correctly by every member of both groups of freshmen. "Clearing of fractions" in No. 24 proved a Waterloo for many who wrote "$3x + 4x = 7$." On the whole, however, even the one-year group did better than the high school group. Evidently college freshmen have learned and remember how to solve simple equations.

Part VII. Special Products

25. $(1 + x)(x - 1) =$
26. $(x - 1)^2 =$
27. $(t + \frac{1}{2})^2 =$

	1 Year Group	1½ Year Group	High School Group
25.	48	83	66
26.	33	89	68
27.	24	86	70

The one-year group shows a very poor retention of the technique of these special forms, but the mastery of the one and one-half-year group is very satisfactory. The high-school group occupy an intermediate position on all three questions.

Part VIII. Factoring

28. $a^3 - 3a^2 =$
29. $16 - 9b^2 =$
30. $5x^2 \div 16x + 3 =$

	1 Year Group	1½ Year Group	High School Group
28.	33	88	68
29.	29	78	68
30.	24	80	62

Results are very similar to those for the special products just given in the previous group, but a little lower in the case of both college groups. There is not a marked difference between the three types of factors, although the results are a little better for the first one, where only a monomial factor is involved.

Part IX. Formulas or Literal Equations

31. Solve $c = 2w - 4$ for w.

32. What is the value of t in the formula $i = prt$, if $p = 200$, $r = 0.04$, and $i = 12$?

	1 Year Group	1½ Year Group	High School Group
31.	43	72	39
32.	52	89	51

Here it may be observed that the results for the one-year group are better than for the high-school group—a fact which is true of all the remaining problems of the test. The one-and-one-half year group handled the problem involving numerical substitution satisfactorily, but leave considerable to be desired in the solution of the formula of problem 31.

Part X. Variation

33. In the equation $xy = 1$, if the value of x increases, then the value of y ——————.

	1 Year Group	1½ Year Group	High School Group
33.	48	89	48

Mastery of this problem on the part of the one and one-half year group is very satisfactory.

Part XI. Verbal Problem

34. A storekeeper buys thermos bottles for \$2.80 apiece. If his overhead is 10 percent of the selling price and if his profit is to be 20 percent of the selling price, at what price should he sell them?

	1 Year Group	1½ Year Group	High School Group
34.	43	72	25

Both college groups show distinct superiority to the high school group in their ability to apply their mathematical knowledge to practical conditions, as far as shown by this one problem.

Part XII. Graph

35. Draw a graph from which you can read the interest on any principal (up to $500) for one year at 4 percent. (Graph paper given.)

	1 Year Group	1½ Year Group	High School Group
35.	29	62	12

Both college groups are distinctly above the average for the high schools, but all three groups are disappointingly low.

Summary

The average percentage of correct responses for the twelve groups of problems is exhibited in the following table.

	1 Year Group	1½ Year Group	High School Group
1. Laws of Exponents	57	87	71
2. Fractions (Algebraic)	30	54	41
Fractions (Arithmetic)	61	82	75
3. Decimals	67	84	69
4. Negative Numbers	66	81	70
5. Substitution	81	98	87
6. Equations	78	90	77
7. Special Products	35	86	65
8. Factoring	29	82	68
9. Formulas or Literal Equations	48	80	45
10. Variation	48	89	48
11. Verbal Problem	43	72	25
12. Graph	29	40	12
Average of 35 Problems	**45.1**	**71.8**	**64.4**
Number problems solved by more than 50 percent of group	19	33	27
Number problems solved by more than 75 percent of group	9	27	10
Number problems solved by more than 90 percent of group	1	7	1

The marked increase in algebraic efficiency resulting from an extra half year of algebra is strikingly shown in the last four lines of this table. The average percentage of correct responses changes from 45 to 72 for the two groups. About one-half of the problems given can be solved correctly by 50% of the one-year college group; three-fourths of the same problems by 50% of the high school group; and almost all of them by 50% of the one and one-half year college group. About three-quarters of the problems can be solved correctly by 75% of the one and one-half year college group; but only about a quarter of them can be correctly solved by 75% of the other two groups. One-fifth of the problems can be solved by over 90% of the one-and-one-half year college group, but there is only one problem of the entire set which can be successfully solved by over 90% of either of the other two groups. This problem is No. 20, the simplest equation given.

The greatest fundamental lacks shown by the application of this test to college students are their disgraceful inadequacy in the handling of simple algebraic fractions, their failure with the principle of subtraction of negative numbers, and their inability to draw a simple graph from a formula. The one-year group also shows poor ability in handling special products and factors.

The most satisfactory results are shown in the solutions of equations, substitution and solution of literal equations, and the handling of negative numbers in addition, multiplication, and division.

Predictive Value

This particular test has a significant but not a high correlation with the achievements of the same group of students in freshman mathematics as indicated by their final grades at the close of the fall term. When computed by ranks (See Kelley, T. L.: Statistical Method, p. 193) it is found that $r = 0.40 \pm 0.06$.

The same group of students were also given the Schorling-Sanford Plane Geometry Test at the opening of college. The correlation between results in the algebra and geometry tests is found by the same method to be $r = 0.56 \pm 0.05$.

RECREATIONAL VALUES ACHIEVED THROUGH MATHEMATICS CLUBS IN SECONDARY SCHOOLS

MARIE GUGLE
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For ages mathematics has had the reputation of being the hardest subject in the curriculum. We have often heard of the mathematics grind. Perhaps, of it, more than of other subjects have pupils asked such questions, as "What is it all about?" or "What is the use of studying algebra?"

Of course, according to the older pedagogy, the harder and more abstract the study, the greater was the resulting mental discipline. Mathematics is a symbolic language and therefore requires the highest type of thinking for its mastery. Since the subject is no longer kept in the curriculum by being a universal requirement, its abolishment has been threatened. Teachers of mathematics, like those of Latin and Greek, have been compelled to justify the retention of their subject in the curriculum.

Let us analyze briefly its relation to the seven Cardinal Principles.

1. Health: Mathematics is not directly connected with the teaching of health, except as the general idea of quantity and the measurement of foods and the proper proportions of necessary elements enter into the health teaching.

2. Command of the Fundamental Processes: The elements of arithmetic are, of course, included in this objective.

3. Worthy Home Membership: For meeting this objective, one naturally thinks of the social studies, literature, music, art, and home economics. But I suggest that the knowledge of the making of budgets and of keeping household accounts would keep many a home from being broken. Therefore a study of such phases of mathematics will meet this objective.

4. Vocation: Mathematics of various sorts has long been recognized as an essential part in the training for many vocations. The pity is that some people would limit that which we teach in the schools to the so-called vocational mathematics in its narrowest sense. In the broader sense this is a worthy objective.

5. Civic Education: This objective deals with the social and personal sides of life, particularly the developments of right civic attitudes and the spirit of co-operation. For this training, mathematics is not so essential, however important it may be in carrying on the business of government.

6. Worthy use of leisure: According to the report on these Cardinal Principles, it is stated that "education should equip the individual to secure from his leisure the recreation of body, mind, and spirit—to utilize the common means of enjoyment." Recreation, amusement, diversion, and play are given in the dictionary as synonyms. Most of us can remember some teachers who would have been shocked at the idea that amusement, diversion, or play should have any consideration in a schoolroom. That was a place for work, the harder and less interesting the better. The kindergarten was known as the play school and was belittled accordingly. Its underlying principle of learning through play has become more universal in its application.

The development of the doctrine of interest and the study of the learning process brought out the fact that children learn more readily the thing in which they are interested and in which they are doing something. In play or recreation there are always both interest and participation. Therefore play or recreation has come to have its rightful place in the classroom.

Some will say, "that is all right to teach primary reading through dramatics, but not mathematics. There is no fun in mathematics." Only the uninitiated say that. Some writers of puzzle pages in the newspapers earn more writing of the fun or play in mathematics than some teachers earn teaching the serious side of the subject.

A little of this phase of the subject may be introduced into the regular classroom. More of it should find its place there. But mathematics has such a wealth of material and the time given in the school program is so limited, that the only way is to organize a mathematics club. It was just such a situation existing in my own classes that caused some of my pupils to ask that we form a club. This was in Toledo, Ohio, in 1913.

In a club there is real, serious work done. But since it is self imposed, it is recreational. A club develops initiative, interest to a high degree, and appreciations that are carried over into the classroom. Its social aspect appeals and through it there is de-

veloped in the pupil the ability to argue and discuss a topic freely and naturally. It gives the pupil a chance to develop a physical and mental poise that is highly desirable.

Play or recreation is a recognized human need for both old and young. Is there anything more pitiable than a child who doesn't know how to play or a man who has been such a grind at his work or business that when he has to take a vacation, he does not know how to play or recreate himself?

This objective, worthy use of leisure, is important to the individual, and is growing more important to the nation as the working hours are reduced and people have more leisure time. If used unworthily, more leisure time spells decadence and disaster for the nation. If used worthily, it means growth and re-creation in body, mind, and spirit.

Too many of our people have a distorted notion of having a good time. To some it means being a spendthrift of time, doing nothing or worse than nothing.

It is truly one of the most important duties of the school to train every pupil to have a proper sense of the value of time, to have a real appreciation of the outdoors, of people, of music, literature, and art, of clean sports, and to have some worthwhile hobby in which he is vitally interested. The many so-called extra curricular or collateral activities of the school tend to meet this objective. Surely the mathematics teachers realize here their wonderful opportunity and responsibility.

The introduction of biographical sketches and bits of the history of mathematics are now used in most text books to vitalize the subject and add interest to the study. The usual classroom procedure is to read the paragraphs, but soon the different men and their contributions become confused in the pupils' minds.

For a Hallowe'en program, one mathematics club had the shades of Pythagoras and Descartes meet as in the next world and enter into an argument as to which one made the bigger contribution to the world when he was on earth. Even the witnesses of this little drama, much less the two actors, will never confuse these two mathematicians or their contributions.

We all recognize the power of school spirit. An intense club spirit may be developed in a mathematics club. Having club colors, pins, and banners are helpful. But a still greater degree

comes from having a club song. Some schools and colleges have a serious school hymn and also a school rally song. Some of our mathematics clubs have found it advantageous to have both types of songs. They add much to the spirit and enthusiasm of the club. The greater the interest and enthusiasm in the members of the club, the more lasting will be its influence and the greater will be the carry-over into the classroom and outside life interests.

The question may arise as to whether the pupils realize and appreciate the recreational side of club activity. We may well let the pupils themselves answer that question. They were asked to write about the benefits of club membership or what the club means to them in order that their replies might help new pupils decide whether or not they should join the club. A few of these pupil responses have been mimeographed for distribution.

7. Ethical Character: The last cardinal principle is ethical character, which is recognized as the paramount objective. How can the teaching of mathematics or the work of a mathematics club have as a by-product the development of ethical character?

The greatest exponent of this type of teaching is none other than David Eugene Smith, the revered dean among teachers of mathematics. All who have been fortunate enough to sit in his classes have been imbued with this idea and spirit. Many are familiar with his "Religio Mathematici." I recommend it to all for reading and re-reading.

The January issue of the MATHEMATICS TEACHER contains an article which deals with this seventh cardinal principle. The discussion is based largely on Dr. Smith's treatise. Dr. Smith connects the idea of the immutability of mathematical laws with that of immortality.

In man's attempt to understand God, his infinite power and his infinite love, man gets his truest conception when he tries to understand mathematical infinity. Mathematics deals with truth, unchangeable and everlasting. And what is religion but a seeking after eternal truth?

A study of nature reveals the most marvelous mathematical relationships. The very planets in the heavens are arranged with their distances apart following a numerical series with astonishing exactness. I refer to Bode's law. The buds on the stems

occur in strange mathematical orders. This plan of leaf distribution is connected with the Summation Series or Fibonnaci Series, whose ratios approach 1.618. For further information on this topic, read the book entitled "Phyllotaxis in Relation to Mechanical Law" by Professor A. H. Church of Oxford.

The late Professor Hambidge of Yale has re-discovered the subtle ratios underlying classic Greek art, which is unequaled in beauty and symmetry. He has given this type the name of dynamic symmetry to distinguish it from the static symmetry of the middle ages.

Snow crystals alone warrant the statement that "God geometrizes." These illustrate static symmetry. But the human body, the butterfly, the iris and all growing plants are based on the more subtle dynamic symmetry. These ratios are the basis of the beauty in nature and in art.

Great spiritual uplift comes from the mere contemplation of these mathematical relationships in nature. A deep respect for such creations begets an abiding faith in the Creator.

The school boy is sometimes puzzled as to the meaning of such queer mathematical symbols as $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, and such terms as diagonal, reciprocal, and extreme and mean ratio. But what respect he would have for these, if he realized that artists use them to make their pictures and designs beautiful; that architects use them to make their buildings attractive. How his respect for these must change to awe, when he realizes that God, the Master Artist, uses them to make the maple and the iris, to make the dragon fly and the butterfly, and even the proportions of his own body, in order that the world may be more beautiful and attractive. The Divine Architect has builded a world marvelous in beauty and harmony through the subtle use of number and space relations. Thus He has tried to reveal to us His beauty and truth through nature, the key to whose mysteries is mathematics.

In our teaching and in our clubs let us try to catch the vision ourselves and through the beauty and truth of mathematics help our boys and girls to see and believe.

INDIVIDUALIZED INSTRUCTION IN GEOMETRY[1]

By MARY A. POTTER
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Even in this great era of road making the royal road to geometry has never been built. For centuries students have been climbing the same steep and narrow paths of reasoning that lead up the hill of knowledge to the stronghold of geometry. For many the going has been hard, some have tripped on the first congruent triangles, some have been lost in the difficult windings of indirect reasoning, some have slid over the precipice of memorizing never to see the grandeurs of the paths of reasoning that lead to their goal. We who are among those who act as guides along this path submit this report of our efforts to assist those under our guidance in making the climb with greater ease and pleasure relying on their own increasing strength.

We had boasted that our course in geometry was a good course in reasoning, we discovered that to many it was a poor course in memory training. For some time we tried different plans to correct this fault with indifferent results until four years ago two of the teachers proposed that we attempt to teach geometry by an individual method of instruction without the use of a formal textbook. The consent of an open-minded principal and superintendent making this experiment possible, we set to work writing what our pupils call a home-made text book which has been since twice revised and perhaps improved.

In form it is a loose leaf notebook of mimeographed sheets. In content it consists of an introduction with the customary statements of definitions, axioms, postulates, constructions and exercises. This is followed by the usual five books of geometry with those theorems and constructions recommended by the College Entrance Board and the National Committee—as our good friends the bookmen always remark. It differs from the traditional text book in that each theorem is presented as an original exercise. Our text book contains only the statement of the theorems beneath which blank space is left for the figure, the "given,"

[1] Read at the Washington Meeting of the National Council of Teachers of Mathematics, Feb. 19, 1926.

and "to prove." The proof also is filled in by the student on a sheet of theme paper facing this mimeographed sheet. You will find distributed through the room copies of a completed theorem showing the typed statement and the parts filled in by the pupils. There are also several notebooks of this year that you may wish to examine.

A separate notebook is provided for each semester. At the end of the semester these notebooks are handed in to the teacher before a grade is given and they are not returned. We like to assist our succeeding classes in keeping honest and conserve for them the joy of making their own explorations. At the beginning of the second semester sheets containing statements of theorems proved in semester I are provided for reference.

We attempt to introduce our pupils to geometry in quite the customary way by discussion of the facts learned in intuitional geometry and a further development of definitions, postulates, and axioms enriched by constructions and numerous sets of exercises. We then present to them a mimeographed copy of those definitions, postulates, and axioms which we have laboriously worked out together. One set of exercises that has been particularly successful as an introduction to formal proof consists of one step exercises whose proof depends upon a definition, axiom, or postulate similar to those exercises given later in the course in which the proof is based upon a previously proved theorem. A problem of this type is:

If angle x is a complement of angle a, and angle x is a complement of angle b, what relation exists between angle a and angle b? Why?

With some labor we get the answer, "Angle a equals angle b, because complements of the same angle are equal."

Having struggled successfully with 25 similar problems and having made up a few of his own, the student is a little less dazed when he encounters the formal proof of the first theorem.

The introduction safely made, we proceed cautiously to develop our first theorem on congruence of triangles. When we've done our best in this development, the pupils are presented with a mimeographed copy of that theorem written out in full and also a second sheet containing the bare statement of the second

theorem on congruence of triangles. With the lone suggestion that this may be proved by superposition, there are at least four explorers in a class that are able to work out the complete proof with no further help. The first year we taught by this method that first original proof gave us a decided thrill.

The class now passes through a transitional period of socialized recitation. After a theorem has been successfully worked out with as little help from the teacher as possible, a pupil demonstrates that theorem at the board before the class, while other pupils jot down comments on his recitation. At the conclusion of his demonstration pupils who wish to do so challenge all or parts of his proof in the usual manner of a socialized recitation, framing their comments whenever possible in the form of questions. We have found this a most important step in developing ability for analytical thinking and for that reason give them definite and rather elaborate training in how to ask a question. Ability to ask a question develops in about this order.

The pupil reaches the first stage when he knows that a certain statement is correct. He arrives at the second stage when he not only knows that a certain statement is correct, but that a second statement perhaps similar to the first is wrong. He now advances to a third stage when he not only is able to declare that one statement is wrong, but is able to compare it with the correct statement and defend his position. We used to think we had reached our goal when our pupils reached this point. But in the fourth stage which is that to which we now try to lead our pupils, he not only knows that the true statement is correct, the false one wrong and why it is wrong, but he is able to ask thought-provoking questions which recognize the difficulty presented and direct the student toward the correct conclusion without leading him to it, as Prof. H. L. Miller of the University of Wisconsin so ably suggests in his "Directing Study."

For example, Ruth who is reciting, says, "The whole is equal to the sum of all its parts." Harry, who is at the first stage of development, knows that the statement is correct and can say, "That's right."

If, however, Ruth had said, "The whole is *greater* than the sum of all its parts," poor Harry would be lost; it sounds all right to him.

But John, in the second stage, knows that the statement is wrong and also knows the correct statement, hence he comments, "Ruth's axiom is wrong. She should have said, 'The whole is *equal* to the sum of all its parts.'" We are now free to choose between the truthfulness of John and the truthfulness of Ruth.

Roy, however, has advanced further—he not only knows why the statement is wrong but can offer arguments to prove his point, so he says, "The whole cannot be *greater* than the sum of all its parts. If that were true, we would never be willing to change a dollar bill for a half and two quarters." Perhaps he would then draw a circle on the board and divide it into parts to show that a whole circle is made up of the sum of all its parts.

But Anne at the ultimate stage, handles the situation quite differently. When Ruth has said, "The whole is *greater* than the sum of all its parts," she would offer no explanation, but begin a battery of questions such as: "How does a dollar bill compare in value with a half dollar and two quarters?"

Ruth replies, "The values are equal."

Anne continues, "If we consider a dollar a whole thing, what would halves and quarters be called?"

"Parts."

Then, "Remembering the dollar and its parts, how do anything and its parts compare?"

Ruth has arrived. "Oh, I see!" and quotes the correct axiom. Often Ruth says, "Oh, I see!" sooner than this.

Of course this method of questioning is as old as Socrates, but its great age makes it no less important.

After this preliminary training of about eight weeks, we are ready for the final form of our work. Later procedure is as follows: Students are given sheets containing the statements of all the definitions, theorems, corollaries, constructions and exercises about a central topic as perhaps, parallel lines. The class in general recitation takes a long-distance view of parallel lines, develops the definition and axiom, learns to locate alternate-interior and other related families of angle pairs, and observes facts that appear to be true about parallel lines and facts that seem to make lines parallel. Standards of attainment for that

unit are then assigned and individual work begins. Each student is held responsible for the proof of each required theorem. They are all attacked as original exercises. A pupil progresses at the rate of speed dictated by his own ability and ambition. He first writes up the proof of a theorem in his note book which is graded by the teacher and then asks to recite it. These numerous recitations are made possible only by the use of student teachers. After a student recites a theorem perfectly he is appointed a student teacher on that theorem, and has other pupils assigned him for recitation. He is given instruction to report errors in proof, not to let the other pupil put anything over on him, and not to leave the theorem until he is sure that his pupil understands it perfectly. With such training capable student teachers become rigid critics and sticklers for detail—they have no heart—they ask searching questions, will bow to the unproved statements of no one as authority and engage in word battles with their pupils over unusual proofs. Meanwhile, the teacher who is paid to teach, acts as general manager, qualifies different student teachers, assigns pupils to their student teachers so that several groups are working at one time, listens in occasionally to see that the student teacher is functioning properly, helps those who are in too great difficulty, and acts as arbitrator of disputes when different methods of proof are offered.

General class recitations are resumed on occasion in three-minute quizzes of fundamental facts to be emphasized such as "Define parallel lines," in drawing difficult figures like the one commonly used for proving that the alternate-interior angles of parallel lines are equal, and in discussing type errors which consume too much time for individual correction such as the fact that a construction line can be drawn to do but one thing, all other properties it may possess are acquired by proof. Drill exercises on parallel lines are worked at any fitting time during the unit of work and are thoroughly discussed in class. About one-third of the time devoted to parallel lines or any other unit of work is spent on such general class recitation and the remaining two-thirds in study and individual recitation.

After the students have finished the proof of the theorems on parallel lines they are given a day for review in which they work

another group of problems which we call exercises for credit. These are solved independently during a specified length of time in a class period. Differences in ability and speed are shown here with startling exaggeration—some pupils who do passing work can solve successfully only three exercises, while others conquer fifteen.

Having demonstrated the theorems and worked exercises, the pupils are now ready for their test on parallel lines which is uniform throughout the department. Summer before last a class in the Teaching of Mathematics at the University of Chicago designed a series of six tests for plane geometry—one for the introduction and one for each of the five books, each test consisting of several parts, such as computation, sentence completion, true-false, and construction. One of our teachers who was a member of this class brought these tests to us and we used them last year. This year we have devised a series of our own of a similar kind, but more difficult and designed to cover each unit of work, such as parallel lines or congruent triangles. The day following the test we discuss with the pupils their errors hoping to strengthen the weak spots in our teaching.

Succeeding units of work go through the same course of treatment given to parallel lines.

The Racine High School is operated on a weighted credit system in which a student who passes in all his subjects, may earn in one year from 32 to 52 hours of credit towards graduation, depending on the quality of his work. Hence it is necessary to make very definite standards of attainment for the different grades the students desire to earn. As I said before, the work is assigned by units, each unit is a topic or a phase of a topic and consists of the theorems, corollaries, exercises, and often constructions on that topic. This unit of work is to be completed within a specified time—a two weeks period is very satisfactory. We then advertise on the blackboard the number of theorems, corollaries, and constructions necessary to earn the various grades. We usually plan that a student who gets the highest grade must do at least twice as much work as the student getting the lowest passing grade.

You say that the bookkeeping for all this is rather complicated? Perhaps, but like many other things, it is a case of getting it down to a system.

Each of us have a slightly different method which doesn't seem to bother us unduly. Mine is as follows: I prepare a card for each unit of work like the paper that I have distributed. It contains at the left the names of the pupils in a class and at the top the numbers of the theorems, corollaries, and constructions in that unit. At the beginning of the class or at any time during the period, pupils may come to my desk and write on a sheet of theme paper their names and the number of the theorem they wish to recite. As soon as possible I either hear them myself or assign them to a student teacher, writing the name of the student teacher after the name of the pupil who is to recite. The student teacher then records after his name the success of the recitation. Completed, each line reads something like this:

Robert Jones, Theorem 8; John Brown O. K., meaning that Robert Jones recited Theorem 8 to John Brown successfully. Some time during the class period I transfer this temporary record to the more permanent record of the card, putting a T for the pupil who becomes a student teacher on that theorem and a check for any other satisfactory demonstration. The paper you have in your hand shows the actual record of my third period class last Tuesday. This card is always on display on the bulletin board. We believe again that it pays to advertise.

This individual method of teaching geometry is now being used in Racine for the fourth year in from six to nine classes each semester with a great variety of students. Those in classes this year vary in age from our infant of 10 to a young man of 22. Although geometry is an elective subject, we have the mine run of students in our classes instead of the screened product. According to a survey of the schools made last year the I. Q.'s of our geometry students vary from 85 to 133, but only 12% can rate 120 or better, more than half are below 110. Since the usual I. Q. for a senior high school student is about 112, I believe, our students are not unusually bright. Our pupils are chiefly foreign born or the children of foreign born parents from southern as well as northern Europe. We have Vertank Gerbankian from

Armenia and Mary Palermo from Sicily, as well as Dagmar Jorgensen from Denmark. Few of their parents have studied geometry, they have not seen geometry books at home, and often late in their course they come to us with the discovery, "There is a geometry book in the public library that has the proof of the theorems written out complete." This is always delivered with a superior air; they seem to wonder that anyone should need so much help.

This method of instruction is not restricted to one teacher, but is used by four teachers, who work together very closely, each keeping a careful notebook and each presenting criticisms and inspirations at a weekly luncheon meeting.

Of course this is a great deal of work for the long suffering teacher. Then why bother? We think it is worth while for many reasons. Our pupils like this method of instruction. Geometry is a popular subject in high school; an election by 80% of the students can be explained in no other way because they do not need credit in geometry for graduation and only 27% of them enter a college or university. Moreover, every year some of my former pupils ask me if they may hear my present class recite theorems after school or in their free periods. This method of instruction is flexible; it lends itself to the ability of the slow student who can usually attack the minimum requirement and feel that he has mastered something, and yet furnishes an incentive to the excellent student by offering him as much work as he is able to do. Such instruction puts the pupil on his own responsibility and develops initiative. We rarely ask a pupil to recite a theorem, he asks us if he may. Formerly the game was to see if he could slip out of a recitation, it now is to see whether he can get a chance to recite. This attitude has led to a happily low percent of failures—less than 5%.

Our geometry courses has been enriched by new proofs for theorems that pupils have evolved. Not being restricted by a book to guide them, they have concocted wonderful proofs sometimes painfully long, but occasionally concise. Often three, sometimes half a dozen proofs are brought in for the same theorem. Most of these proofs we have found in some text, but a few are new to us and we can see nothing wrong with them.

It has been a rather humilitating experience to us who have taught geometry from a book for many years previous to find out how little geometry we really know. When we have no sustaining prop of a text for authority we have had to search diligently through many texts, have argued fervently with each other and with our pupils, and have learned more geometry that we knew existed—enough that we now are very modest of our small store of knowledge.

Constant repetition makes the course thorough. The pupils cannot readily forget facts they have heard so frequently repeated.

Because the theorems are presented somewhat in the form of an outline we find pupils organizing facts more intelligently about a central topic. For instance, some weeks after we have finished the subject of parallel lines they are able to state the facts about parallel lines with very little review and usually remember whether a theorem belongs under the heading of parallel lines or application of parallel lines.

Moreover, we teachers like this method of instruction. At the end of each year we have frankly discussed returning to our old way of teaching, and every year we have unanimously decided to continue because we feel so much better satisfied with our pupil's grasp of the subject. Our teacher judgment was confirmed by the record made by our pupils in the Schorling-Sanford Geometry Tests given last June as part of a city survey in which more than three-fourths of our pupils exceeded the standard median. This test was given again to the 72 pupils who finished geometry the first semester of this year, and only one of them fell below the standard median. Although the price in labor is high, we are sold to an individualized method of instruction in geometry because of the increased dividends.

THE "ATTACK" IN PROPOSITIONS ON INEQUALITY OF LINES

By ARTHUR HAAS
Thomas Jefferson High School, New York City

After experiencing trouble in teaching propositions involving inequalities, especially the one in which two triangles were involved, and after having noticed, term after term, the hopeless floundering of pupils endeavoring to work out original exercises which required the proof that one line was longer than another, the writer undertook devising a general method of approach which would aid the student in his endeavors along these lines. The result, year after year, showed such improvement, that the writer feels justified in offering it to others who may have suffered from the same trouble, in the hope that it will prove of equal value to them.

Axioms Involved

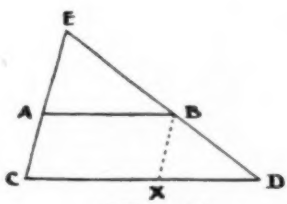
1. The whole is greater than any of its parts.
2. If equals be added to unequals the sum will be unequal in the same sense.
3. If equals be taken from unequals the remainders will be unequal in the same sense.
4. If unequals be taken from equals the remainders will be unequal in the opposite sense.
5. If three things are so related that 1st $>$2nd, and 2nd $>$3rd, then the 1st $>$the 3rd.
6. A straight line is shorter than a broken line joining its extremities. (The shortest distance between two points is measured on a straight line.)

The first of these is invoked where the two lines can be made to coincide in *one* extremity and in direction.

As an illustration of this, the theorem, "A line limited by two sides of a triangle and parallel to the third side is shorter than that side."

Here the line is moved parallel to its original position so that the extremity "*A*" remains in line *AC* until it reaches point

"*C*." The extremity "*B*" will, of course move in a line parallel to line *AC*, away from line *BD*. *ABXC* will be a parallelogram, and



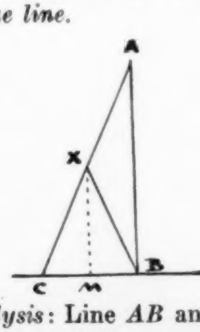
$$CX = AB$$
$$CX < CD$$
$$\therefore AB < CD$$

The second, third and fourth axioms are purely algebraic and are used in connection with other proofs; but the fifth is the one that can be used in nearly every proposition and, if properly introduced, can be made a very real instrument for original exercises involving inequalities.

It may be stated in the analysis as follows. If two lines are made to coincide in their extremities, the one that breaks is the greater; and the problem for the pupil is to find the breaking point.

The application of this principle will be gradually unfolded as the ensuing propositions and exercises are explained.

I. Theorem—If from a given point outside a line, a perpendicular and oblique line be drawn, the perpendicular will be shorter than the oblique line.



Discussion and Analysis: Line *AB* and line *AC* have "*A*" in common. Where can we break line *AC* so that "*C*" will fall on "*B*" without disturbing the rest of the line *AC*.

Obviously, by bisecting line BC and drawing a perpendicular at the mid-point "M," we get a point "X" (in AC) which is equally distant from "C" and "B."

Hence by breaking AC at "X" and bringing the line segment XC to the position of XB we have the broken line AXB = the straight line AXC and greater than straight line AB. (Axiom "6" above.)

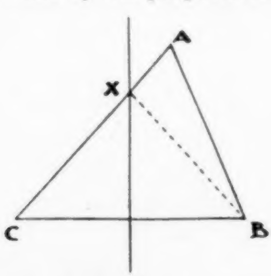
$$\therefore AC > AB \text{ substitution.}$$

Note—We could find "X" likewise by constructing $\measuredangle XBC = \measuredangle XCB$ within $\measuredangle ABC$.

The formal proof involving this is obvious.

$CX = XB$
∴ Broken line AXB = Straight line AXC
but " " $AXB >$ " " AB
∴ $AC > AB$

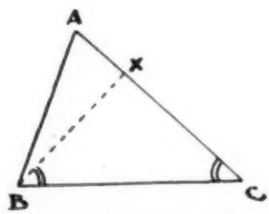
II. Theorem—A point not on perpendicular bisector of a line is unequally distant from its extremities and is nearer that end which lies on the same side of the perpendicular.



Discussion and Analysis: The two lines AB and AC have point "A" in common. The problem again is to find a breaking point in AC, so that without disturbing part of the line we can bring the point "C" over to "B." Obviously "X" is the point; (the contradictory of our proposition, yielding the required authority).

$XC = XB$
∴ AXB (the broken line) $= AC$
But $AXB > AB$
∴ $AC > AB$

III. Theorem—If two ∢s of triangle are unequal then sides opposite are unequal, that side being the greater which lies opposite the greater ∢.



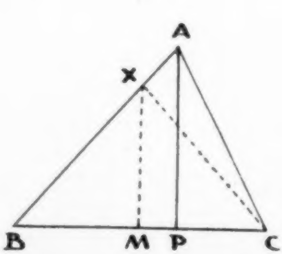
Discussion: Again the two lines have point "*A*" in common, and the problem is to find a point in the longer line which is equally distant from the remaining extremities of the shorter.

This is done by using the contradictory of the proposition to be proved; that is, if base angles are equal, the sides opposite are equal.

We therefore construct ∢3 = ∢"*C*"

$\therefore XC = XB$
AXB (the broken line) $= AXC$ (the straight line)
$AXB > AB$
$\therefore AC > AB$

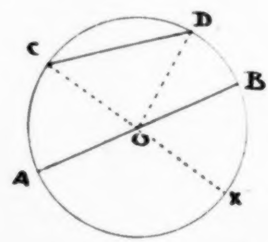
IV. Of two oblique lines drawn from a point to a line from a point without the line, that is the greater which meets the line at a greater distance from the foot of that perpendicular to the line drawn from the original point of intersection of the two lines.



Discussion: Of course we can involve No. 3 after showing that ∢ "*C*" > ∢ "*B*," or again No. 2 by showing "*A*" is not on the perpendicular bisector of *BC*.

But it is just as easy to show that since "P" is not mid-point of AB, $\therefore$ a perpendicular to AB at "M" will cut line AC in a point equally distant from B and C and then the proof becomes exactly of the same type as in the previous examples.

V. A diameter is > any other chord.



Discussion: Here neither extremities coincide. Therefore we must move line AB to some position CX, so that point A falls on point C; then find a breaking point in line CX which satisfies the condition of being equally distant from D and X.

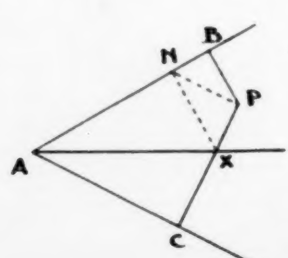
This is obviously the center of the circle and again after necessary construction we have

Broken line $COD =$ Straight line AOB

Broken line $COD >$ Straight line CD

$\therefore \quad AB > CD$

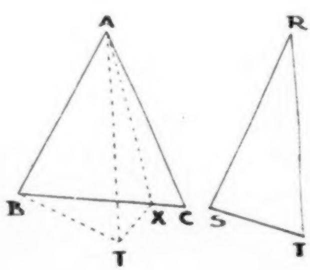
VI. Any point not on bisector of $\sphericalangle$ is unequally distant from the sides of that $\sphericalangle$.



Discussion: $NX = CX$ (contradictory of proposition to be proved)

$\therefore$ Broken line $NXP =$ Line CXP	$= s + = s = = s$
Broken line $NXP >$ Line PN	(Ax. 6 above)
$\therefore CP > NP$	Substitution
Also $NP > PB$	(Proposition 2 above)
$\therefore CP > PB$	(Ax. 5 above)

VII. If in two triangles, 2 sides of one = 2 sides of other but included angle of 1st > included angle of 2nd, then the 3rd side of the 1st > 3rd side of 2nd.



Discussion: Shows that first we must move line RT so that R falls on A and S falls on B.

This is done by making triangles coincide as far as possible, that is, line RS falls on equal side AB and line AT falls between lines AB and AC.

To find point on line BC equidistant from points T and C, we invoke contradictory of proposition. That is, $SAS = SAS$.

This is done by using the equality of lines RT and AC, the identity of a construction line and the equality of two angles, just as in proof of theorem that base angles of isosceles triangles are =. Point "X" is found on the bisector of angle A.

Note (*a*) **Again we find equality of lines XT & XC by using the proposition of which this is the contradictory.**

Note (*b*) **Many pupils in class have suggested that line CT be drawn and its perpendicular bisector used to find point "X"; and here we must show that the error comes from not using that part of the hypothesis which states that line RT = line AG; and that there is no way of showing that perpendicular bisector of line TC might not fall outside of line BC. (Prolong line AT till perpendicular bisector of CT falls in front of point "B" to illustrate this point):**

Proof

$XT = XC$		Hom. Parts.
Broken line	$BXT =$ Straight line BXC	$= s + = s$
But " "	$BXT > BT$	Axiom 6 substitution
Whence	$BC > BT$	
And	$BC > ST$	

By developing the rationale of this theorem, we avoid the common error of saying that Line BT + Line TX > Line BX; which, while true, leads us nowhere.

Many exercises involving inequalities may be given, and if the fundamentals are well understood they will yield neat, elegant and satisfactory proofs to the student. Among them are the following:

1. Prove: A tangent is *longer* than any other line drawn from a point outside a circle to meet the convex curve.
2. That tangent is *shorter* than any other drawn from a point to meet the convex arc of a circle.
3. That any line from the vertex of an isosceles triangle to a point on the base is less than either side.
4. That any line drawn from the vertex to the base, prolonged, is greater than a side.
5. That if base $\measuredangle$s of trapezoid are unequal, sides opposite are unequal and the greater side is adjacent to the smaller angle.
6. That if two unequal circles intersect then that radius is the smaller, which makes the smaller $\measuredangle$ with the common chord, etc.

AN ELECTIVE COURSE IN MATHEMATICS FOR THE ELEVENTH AND TWELFTH SCHOOL YEARS[1]

By GORDON R. MIRICK and VERA SANFORD
The Lincoln School

The purpose of this article is to describe the content of the mathematics work of the eleventh and twelfth years as it is now being given in the Lincoln School. No attempt will be made to compare it with other work for the same years now in progress elsewhere.

The course is designed to replace the one semester units of intermediate algebra, trigonometry, solid geometry, and advanced algebra which, with various permutations of the last three subjects, is the customary offering of the last two years of the high school. It is felt that work arranged on a year's basis can be more economical of time and that it can present a truer conception of mathematics as a unified body of thought.

The inadequacy of the one semester arrangement in the colleges has resulted in the development of general courses for the freshmen, providing greater appreciation of the nature and usefulness of mathematics and an earlier introduction to concepts once reserved for later years.[2]

It is true that the texts used for the one semester courses in the colleges are frequently taught with success in the high schools. It has been the experience of the writers, however, that the works written for the unified freshmen courses in college are too condensed to be used as the sole texts in an eleventh grade class. Furthermore, such books frequently assume knowledge which the student has not acquired at this time. The use of these texts, supplemented by more elementary material, on the other hand, gives convincing evidence of the possibility of teaching the subject matter to pupils of these classes. In the twelfth grade, the case is somewhat different, and such works may be used as the

[1] The substance of this paper was given at the Washington meeting of the National Council of Teachers of Mathematics, Feb., 1926.

[2] Vera Sanford, "Textbooks in unified mathematics for college freshmen," *Mathematics Teacher*, Vol. XVI; pp. 206-215.

sole textbook with profit, the reason being that the pupil who elects this work is generally particularly apt in mathematics and he is likely to be superior in this regard to many of the students who study the subject as college freshmen.

The purpose of the course is to offer elective work, to which a year of plane geometry is prerequisite, with subject matter that is sound from the standpoint of the mathematician and useful for the citizen whose interests make him wish a broader knowledge of the subject than his constant needs require. Since this work is to be elective, it is necessary that the first year be a unit in itself, for many students will limit themselves to this year only. During the time in which this course has been in operation, the pupils planning to enter college with advanced credit have followed both years of the sequence. Those offering only elementary algebra and plane geometry have fulfilled that requirement by the end of the eleventh grade work.

A detailed statement of the content of these courses is given at the close of this article, but certain sections require a preliminary word of explanation.

The Eleventh Grade Course

The review of ninth year algebra at the beginning of the eleventh grade is given with a constant effort to make the student appreciate the logic that is involved. The review thus accomplishes a dual result: bringing back to the pupil's mind the work he has probably forgotten, and giving him a deeper conception of its significance.[1]

The dominant idea of the year's work is the function concept,—a choice which may be justified both by the universality of its use in science and in business and also by its importance in later work in mathematics. Certain mathematical terms descriptive of different types of variation are discussed during the study of graphs and statistical tables, with the purpose of equipping the student with a vocabulary that will permit the concise and clear summarizing of them.

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[1] See Gordon R. Mirick and Vera Sanford, "A study of a pupil's knowledge of algebra at the beginning of his junior year in high school," *Mathematics Teacher*, Vol. XVIII; pp. 171-181.

In the treatment of linear functions, work in the analytic geometry of the straight line is introduced. As a result, the pupils are able to derive the mathematical formulas, as for example Hooke's Law, that connect empirical data obtained from experiments performed in class.

The work in trigonometry includes the logarithmic solutions of right triangles and of triangles which may be solved by the Law of Sines. This topic is connected with the theme of the year's work through the illustration of periodic functions brought out in the study of the general angle. The other topics of the traditional course in trigonometry are postponed until the senior year.

A short unit in mechanics is introduced for the purpose of showing the application of trigonometry to problems involving quantities that have direction as well as magnitude.

The discussion of quadratic functions includes the algebraic and graphical treatment of quadratic equations. The locus work of plane geometry is extended by considering the parabola, circle, ellipse, and hyperbola as loci.

This work occupies about two-thirds of the year,—a time-schedule made possible by the subordination of topics whose sole use lies in solving intricate and improbable examples, and by the economy resulting from the close organization of the work.

The remainder of the year is given to a unit of solid geometry for the purpose of strengthening the pupil's concept of spatial relations and enabling him to prove familiar formulas regarding areas and volumes. The traditional work of Book VI is given with particular emphasis on the idea of orthogonal projection. An algebraic approach to certain theorems regarding areas and volumes is made possible by the use of the formula for the sum of the squares of the first n integers, which has been derived previously by mathematical induction. In a number of theorems, the ideas of the summation of small elements and the limit of a sum are involved, and thus the student receives an informal introduction to the notion of the calculus without using any of its formal technique. The course omits the topic of spherical geometry but includes, for its informational value, a discussion of different types of projection as applied to map-making.

The Twelfth Grade Course

The order of work in the twelfth year is somewhat governed by the fact that many of the students expect to take a college entrance examination in advanced mathematics in the spring. Accordingly, the last half of the year is devoted to advanced algebra and to completing the work in trigonometry. The content of this work is largely that stated in the requirements of the College Entrance Examination Board for this subject (Mathematics Cp. H),[1] but the technique acquired in the first half year is used whenever possible.

The key idea of the first half year's work is the extension of the function concept to that of the rate of change of a variable quantity. This leads directly to the introduction of differentiation and to its applications to mechanics in velocities and accelerations. The indefinite integral as the inverse of the derivative, is used both in the computation of more intricate areas and volumes than those treated at the close of the eleventh grade, and also in the solution of problems dealing with momentum, force, and work.

Eleventh Grade Course of Study

I. Introduction:
  1. Classification of operations;
  2. Need for the introduction of fractions and of negative, irrational, and imaginary numbers into our work in mathematics.
  3. Direct operations limited to simple cases. Special products. Under this work the fundamental laws of algebra will be considered (i.e., communicative, associative, distributive, $a^m \cdot a^n = a^{m+n}$, etc.).
  4. Simple and fractional equations. Problems.
  5. Motion problems. Algebraic and graphical solution.
  6. Notion of functional relationship.

II. Inverse operations:
  1. Factoring—limited to the following cases: common monomial factor, $a^2 \pm 2ab + b^2$, $a^2 - b^2$, $ax^2 + bx + c$.
  2. Fractions—limited to simple cases.
  3. Equations and the transformation of formulas.
  4. Problems.

[1] College Entrance Examination Board, Documents 107 and 108.

III. Functions:

1. Informal discussion.
2. Means of representation and the advantages and disadvantages of each.
3. Properties of functions.
   a. Increasing and decreasing.
   b. Continuous and discontinuous.
   c. Single and multiple valued—periodic.

IV. Linear Functions and Simultaneous Linear Equations:

1. Properties of a straight line.
2. The derivation of the type equations of a straight line. (This necessitates a review of the definition of a tangent and the extension of this definition to angles greater than 90°.)
3. Algebraic solution of simultaneous linear equations.
4. Problems.

V. Transcendental Functions:

1. Exponents and exponential functions.
2. Logarithms, their application to growth curves and to compound interest.
3. Trigonometric functions of the general angle.
   a. Solution of right triangles by natural functions and by logarithms.
   b. The Law of Sines.
   c. Application of trigonometry to surveying and mechanics.

VI. Quadratic Functions:

1. Graphical solutions.
2. Algebraic solutions—radicals, imaginary numbers.
3. Elementary properties of the following loci:
   a. Parabola.
   b. Circle.
   c. Ellipse
   d. Hyperbola.
4. Extension of the locus work of plane geometry by means of this work in analytics.

VII. Progressions—Problems.

VIII. Mathematical Induction:
Binomial Theorem—Problems.

IX. Solid Geometry:
1. Book VI with special emphasis on projection.
2. Area and Volumes. Proofs more algebraic.
3. Stereometry—Map Projection.

Twelfth Grade Course of Study

I. Review of Progressions.

II. Mechanics—The consideration of the notion of velocity and acceleration.

III. Functions and their graphs.—Variation.

IV. Differentiation of Algebraic Expressions:
1. Fundamental formulas.
2. Maxima and minima.
3. Applications to mechanics.
4. Small errors—measurement.

V. Integration of Simple Algebraic Expressions:
1. Elementary notion—indefinite integral.
2. Applications.
a. Areas and volumes.
b. Mechanics—force, work, etc.

VI. Imaginary and Complex Quantities:
1. Rectilinear and Polar Representation.
2. De Moivre's Theorem.

VII. Theory of Equations (some use is made of the calculus).

VIII. Determinants.

IX. Trigonometry completed.

X. Scales of Notation.

XI. Permutations Combinations.
1. Probability.
2. Statistics.

Bibliography

The following books and articles have been of especial assistance in this study:

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2. G. A. Bliss, "The function concept and the fundamental notion of the calculus," in *Monographs on Modern Mathematics,* edited by J. W. A. Young, Longmans, Green, & Co., 1915.
3. Gale and Watkeys, *Elementary Functions and Applications,* Henry Holt and Company, 1920.
4. F. S. Griffin, *Introduction to Mathematical Analysis,* Houghton Mifflin, 1922, Chapters I-IV.
5. H. E. Hawkes, *Higher Algebra,* Ginn and Company, 1913, especially pp. 23-29.
6. W. R. Longley, "Some limit proofs in solid geometry," *American Mathematical Monthly,* Vol. XXXI, pp. 196--202.
7. T. Percy Nunn, *The Teaching of Algebra including Trigonometry,* Longmans, 1914; pp. 386, 387, 441-453. (map-making).
8. J. B. Reynolds, "Some application of algebra to theorems, in Solid Geometry," *Mathematics Teacher,* Vol. XVIII, pp. 1-9.
9. E. J. Townsend, *Functions of a Complex Variable,* Henry Holt and Company, 1915, especially pp. 1-5.

MINUTES OF THE MEETING OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS HELD IN THE RALEIGH HOTEL SATURDAY, FEB. 20, 1926

A. The morning session was called to order at ten o'clock by President Raleigh Schorling.

The minutes of the last meeting were read by the Secretary and after correcting the list of names of last year's auditing committee, the minutes were approved.

President Schorling laid before the meeting certain data relating to the Yearbook, giving the cost to the Council of editions of various sizes. An edition of 2,500 copies would cost $925.25; of 3,000 copies, $1,028.69, and of 3,200 copies, $1,070.04. Eight hundred and ninety-two copies have already been applied for by various persons and institutions.

After stating the facts upon which the members present would base a decision as to the size of the edition to be taken by the Council, the regular program for the meeting was taken up.

Miss Annabel White of Baltimore reviewed Professor David Eugene Smith's paper in the Yearbook.

Mr. William Betz reviewed and commented upon his paper in the Yearbook on The Development of Mathematics in the Junior High School. Mr. Betz discussed the history and development of the junior high schol as an institution, reserving his comments upon the mathematics of the junior high school program of studies until a later session.

At the close of Mr. Betz's remarks, the President ordered an intermission taken.

Mr. George W. Evans read his paper on Orthodoxy and Heresy in the teaching of geometry.

The President appointed the following auditing committee: Miss Gugle, Mr. Winter, Mr. Marquard.

The President appointed the following committee to canvass the votes cast for officers for the coming year: Miss Worden, Mr. English, Mr. Foberg.

The morning session then adjourned.

B. At the noon luncheon of the executive committee, the President invited in a number of members of the Council to confer with the Executive Committee. The following action was taken by the Executive Committee:

1. It was decided to recommend to the Council that 3,200 copies of the Yearbook be ordered from the publishers.

2. It was decided to recommend to the Council that the price of the Yearbook be fixed at $1.00 per copy, plus expressage charges, for lots of 20 or more, and for smaller lots, $1.10 per copy, postage prepaid.

3. It was decided to recommend that a vote of thanks be extended by the Council to Mr. C. M. Austin for his willingness to assume the responsibility of handling the business affairs connected with the sale of the Yearbook.

4. It was decided to authorize the Yearbook Committee the use, not to exceed fifty copies, of the Yearbook for publicity purposes, and to present one copy to each contributor and to those taking part in the present program.

5. It was decided to recommend a vote of thanks and appreciation to President Schorling for his work in connection with the Yearbook.

6. It was decided to recommend that a vote of thanks and appreciation be extended to Dr. John R. Clark for his work in connection with Mathematics Teacher.

C. The afternoon session convened at 2:15 P. M. Miss Mary Potter read her paper on Individualized Instruction in Geometry.

Professor W. D. Reeve then commented on his paper on Improvement of Algebra Tests in the Yearbook.

Dr. Clark laid before the meeting the following statement concerning the finances of Mathematics Teacher:

"February 19, 1926.

"The National Council of Teachers of Mathematics,
Mr. Raleigh Schorling, President,
Mr. John A. Foberg, Secretary-Treasurer.

Gentlemen:

"On February 18, 1925, I submitted the financial statement for the year ending on that date. During that year there was a credit to the Council of $709.25 and a charge of $1,034, leaving a balance due me of $324.75.

"Since February 18, 1925, I have, up to date (February 18, 1926), received from subscriptions to the Mathematics Teacher $6,208.15. In accordance with the conditions under which I assumed the business management of the Teacher, 12½% of this amount ($776.02) is due the Council. I am submitting herewith a check for $451.27, the net amount due the Council ($776.02 less $324.75).

"The circulation of the Mathematics Teacher has increased slightly during the past year. Continued efforts on the part of those interested in the National Council should increase considerably the circulation during the coming year. I hope that $1,000 or more may be due the Council at the next accounting.

Respectfully submitted,

(Signed) JNO. R. CLARK."

Dr. Clark stressed the importance of co-operative efforts by all members of the National Council to increase the snubscription list of Mathematics Teacher.

The recommendations of the Executive Committee were laid before the meeting and it was decided to purchase an edition of 3,200 copies of the Yearbook. The other recommendations of the Executive Committee as listed above were taken up one by one, and adopted.

On motion of Dr. Reeve, it was decided that the President is to appoint a necrology committee to report at the next annual meeting.

The Auditing Committee then made its report as follows:

"February 20, 1926.

"The Auditing Committee have examined the books of the Secretary-Treasurer and find them correct to date.

Balance	$135.41
Check not in No. 48	8.00
Bank Balance	$143.41

(Signed) MARIE GUGLE,
" O. WINTER,
" C. B. MARQUARD,
Auditing Committee."

On motion, the report was accepted as read.

Miss Worden reported for the committee appointed to count the ballots as follows:

Total number of votes cast, 106.

For President, Miss Marie Gugle, 55 votes.
Mr. Wm. Betz, 51 votes.

For Vice President, Mr. W. W. Hart, 82 votes.
Dr. E. M. Taylor, 24 votes.

For members of the Executive Committee,
Dr. W. D. Reeve, 93 votes.
Dr. F. Touton, 51 votes.
Dr. E. B. Lytle, 35 votes.
Dr. W. C. Eels, 26 votes.

(Signed) ORPHA E. WORDEN,
HARRY ENGLISH,
J. A. FOBERG.

The President then declared Miss Gugle elected President for the next year; Mr. W. W. Hart, Vice-President; and Dr. Reeve and Mr. Touton members of the Executive Committee, with terms to expire in 1929.

Mr. Gordon R. Mirick then read a paper on Elective Courses in Senior High School Mathematics. At the close of this paper, the meeting adjourned.

D. At six o'clock the dinner of the National Council was held in the Raleigh Hotel.

Superintendent Frank Ballou of Washington, D. C., delivered an address of welcome.

Dr. Harvey Wiley discussed the metric system.

Miss Marie Gugle, President-elect of the Council, read her paper on Recreational Values Achieved Through Mathematics Clubs in Secondary Schools.

Mr. William Betz completed his discussion of Mathematics in the Junior High School.

President Raleigh Schorling delivered his Presidential Address, which was distributed to the members present in printed form.

A cordial vote of thanks and appreciation was extended to the local committee of arrangements for the Washington meeting. The meeting was then adjourned.

The following persons attended the meeting:

NAME SCHOOL CITY

Albert, Gertrude, Central High School, Washington, D. C.
Allshouse, Margaret O., Hood College, Frederick, Md.
Anderson, Helen C., Eastern High School, Washington, D. C.
Arnold, Katherine, Hood College, Frederick, Md.
Atwood, N. M., Shaw Junior High School, Washington, D. C.
Ballard, Mrs. Sargent I., Central High School, Washington, D. C.
Bastian, Alice, McKinley Technical High, Washington, D. C.
Bechtel, John C., Germantown High School, Philadelphia, Pa.
Betz, William, Junior and Senior High Schools, Rochester, N. Y.
Birtwell, Bertha, McKinley Technical High, Washington, D. C.
Blandford, A. S., Wallach High School, Washington, D. C.
Blandford, Mary K., Western High School, Washington, D. C.
Bowers, Jacob, Northwood School (Prin.), Columbus, Ohio.
Calloway, Caroline C., Dunbar High School, Washington, D. C.
Campbur, Anna, Jefferson Junior High, Washington, D. C.
Clark, John R., The Lincoln School, New York, N. Y.
Clements, J. R., Sterling Junior High, Columbus, Ohio.
Coombs, D. S., Central High School, Washington, D. C.
Cornell, Florence N., Columbia Junior High, Washington, D. C.
Cromwell, Mary E., Dunbar High School, Washington, D. C.
Crook, E. M., Powell Junior High, Washington, D. C.
Dean, May O., Central High School, Washington, D. C.
Ebaugh, Harriett, McKinley Technical High, Washington, D. C.
Edmondson, Jessie B., Western High School, Washington, D. C.
English, Harry, Franklin High School, Washington, D. C.
Ernst, Ursula A., York High School, York, Pa.
Evans, Geo. W., (Retired), Boston, Mass.
Evans, Harriet, Langley Junior High, Washington, D. C.
Farr, S. M., Central High School, Washington, D. C.
Fink, A. R., York High School, York, Pa.
Flowers, Ida V., Montbello High School, Baltimore, Md.

Foberg, J. A., State Dept. of Public Instr., Harrisburg, Pa.
Frankenfield, M., Hine Junior High, Washington, D. C.
Gamper, Hedwig, Everett Junior High, Columbus, Ohio.
Gibbs, K. M., Langley Junior High, Washington, D. C.
Gilbert, L. E., Central High School, Washington, D. C.
Goodman, Alma B., Macfarland Junior High, Washington, D. C.
Grim, T. H., York High School, York, Pa.
Gugle, Marie, Ass't Superintendent, Columbus, Ohio.
Hainor, Pearl, Lincoln Junior High, Huntington, W. Va.
Hammond, Ida, McKinley Technical High, Washington, D. C.
Harmel, Ida, Jefferson Junior High, Washington, D. C.
Harris, Ethel C., Armstrong Technical High, Washington, D. C.
Hart, Sara T., Jefferson Junior High, Washington, D. C.
Hart, Walter W., Wisconsin University, Madison, Wis.
Haynes, E. L., Dunbar High School, Washington, D. C.
Henry, Etta M., High School, Kenova, W. Va.
Hobgood, K. W., Macfarland Junior High, Washington, D. C.
Hodgkins, G. W., Central High School, Washington, D. C.
Howard, Y. F., Director Junior Schools, San Antonio, Tex.
Huntzberger, I. W., Central High School, Washington, D. C.
Johnssen, Edna, Western High School, Washington, D. C.
Kellerman, Gertrude H, Central High School, Washington, D. C.
Kennedy, Julia E., (Grad. Student) Trinity College, Washington, D. C.
Kennedy, Kathryn O., Normal Training High School, Terre Haute, Ind.
Kinnear, Agnes I., Langley Junior High, Washington, D. C.
Kley, Mrs. A. L., Washington, D. C.
Klips, Marion, Mt. Vernon Seminary, Washington, D. C.
Kuman, Ada Bell, Deering High School, Portland, Me.
Kupfer, Julie A., Langley Junior High, Washington, D. C.
Lampert, Blanche, Powell Junior High, Washington, D. C.
Lantenschlager, May, Powell Junior High, Washington, D. C.
Laughlin, B., Parker High School, Chicago, Ill.
Leopold, Marian E., Kensington High School, Philadelphia, Pa.
Lord, W. L., Woodberry Forest School, Woodberry Forest, Va.
MacCormick, Donald E., William Penn Charter School, Philadelphia, Pa.
Macfarren, S. J., Washington, D. C.
Manning, Annie Lee, Montebello High School, Baltimore, Md.
Marquard, C. B., Indianola, Columbus, Ohio.
Marquard, Helen M., Mound Junior High, Columbus, Ohio.
McGrath, Catherine M., Eastern High School, Washington, D. C.
McKnight, N. J., Western High School, Washington, D. C.
McNally, J. V., Sherrard Institute School, Detroit, Mich.
Milk, Dudley, Chicago, Ill.
Miller, Florence Brooks, Fairmont Jr. H. Training School, Cleveland, Ohio.
Miller, H. Brown, Fairmont Jr. H. Training School, Cleveland, Ohio.
Mirick, G. R., The Lincoln School, New York, N. Y.
Mitchell, H. F., Technical High School, Washington, D. C.
Moriorty, Sarah C., (Student) Trinity College, Washington, D. C.
Packer, Margaret C., Hood College, Frederick, Md.
Percival, W. I., Teachers College, New York, N. Y.
Porter, May N., White Plains High School, White Plains, N. Y.
Potter, Mary A., Racine High School, Racine, Wis.
Potts, C. R., Western High School, Washington, D. C.
Powell, Edith Belle, Ross High School, Washington, D. C.
Preston, Amy F., Roosevelt Junior High, Columbus, Ohio.
Readcliffe, Sarah B., McKinley Technical High, Washington, D. C.
Reeve, W. D., Teachers College, Columbia Univ., New York, N. Y.
Roberts, Gertrude, Junior High School, Huntington, W. Va.
Robinson, M. Opal, Handley High School, Winchester, Va.

Rogers, F. W., Port Jervis High School, New York.
Ross, George A., Central High School, Washington, D. C.
Rucker, Ruth E., Columbia Junior High, Washington, D. C.
Richmond, Susan V., Western High School, Washington, D. C.
Sanford, Vera, The Lincoln School, New York, N. Y.
Sauble, Irene, Ass't Supervisor Exact Sciences, Detroit, Mich.
Scarborough, Mary H., State Normal School, Towson, Md.
Schorling, Raleigh, University High School, Ann Arbor, Mich.
Schroing, Antoinette, Hammer High School, Washington, D. C.
Sheffey, Georgie E., Johnson Randall Junior High, Washington, D. C.
Shelp, Gertrude A., Eastern High School, Washington, D. C.
Shenton, Walter F., American University, Washington, D. C.
Shipley, C. H., Macfarland Junior High, Washington, D. C.
Smith, Helen Y., York High School, York, Pa.
Sr. M. Cecilia, S. N. D., Trinity College, Washington, D. C.
Sr. Marie Clara, Trinity College, Washington, D. C.
Stevens, E. N., Ginn & Co., 15 Ashburton Pl., Boston, Mass.
Stickel, R. L., Handley High School, Winchester, Va.
Stritzinger, Marylew, Haverford Twp. High School, S. Ardmore, Pa.
Strong, Theodore, The Park School, Baltimore, Md.
Tennyson, J. Anna, Langley Junior High, Washington, D. C.
Thomas, J. J., Central High School, Washington, D. C.
Thompson, Evelyn R., Western High School, Washington, D. C.
Tipps, Irma, Colonial School, Washington, D. C.
Vaince, Frances M., (substitute) Powell Junior High, Washington, D. C.
Wallis, William J., Central High School, Washington, D. C.
White, Annabel, Western High School, Baltimore, Md.
Wilcox, Charles C., Supervisor, Junior High School, Kalamazoo, Mich.
Wilder, Jennie T., Armstrong Technical High, Washington, D. C.
Wilkinson, Gladys A., Dunbar High School, Washington, D. C.
Wilson, Angeline, Central High School, Grand Rapids, Mich.
Wilson, Elizabeth W., Central High School, Washington, D. C.
Winter, O., Whitney School, Chicago, Ill.
Wolfenbarger, Floy, Jefferson High School, Washington, D. C.
Worden, Orpha, Teachers College, Detroit, Mich.
Yohn, Lottie I., Hood College, Frederick, Md.

IMPORTANT ANNOUNCEMENT

The First Yearbook issued by the National Council of Teachers of Mathematics is now ready for distribution.

Table of Contents

Every progressive teacher of mathematics will want a copy, so send in your orders early.

The price is $1.10 per copy, prepaid in lots of less than twenty copies; in lots of twenty or more, $1.00 per copy, plus express or freight charges. Purchasers will confer a great favor by sending the money in advance.

Address all orders to the Chairman of the Yearbook Committee: Charles M. Austin, High School, Oak Park, Illinois.

NEW BOOKS

A Diagnostic Study of the Teaching Problem in High School Mathematics. By William David Reeve. Ginn and Company, 1926. Price 84c.

The author discusses the uses and limitations of the various types of standardized and objective tests in high school mathematics, and shows how he designed diagnostic tests for the various chapters in the Schorling-Reeve General Mathematics. A non-technical and readable discussion of tests.

Advanced Calculus. By F. S. Woods. Ginn and Company, 1926. Price $4.60.

A book for the second course in calculus which is intended for the use of students who expect to enter into engineering or scientific work. Under chapter headings we have: Power Series; Partial Differentiation; Implicit Functions; Applications to Geometry; The Definite Integral; The Gamma and Beta Functions; Line, Surface and Space Integrals; Vector Notation; Differential Equations of the First Order; Differential Equations of the Higher Order; Bessel Functions; Partial Differential Equations; Calculus of Variations; Functions of a Complex Variable; and Elliptic Integrals. The typography is good and there are an abundance of problems.

GORDON R. MIRICK.

NEWS NOTES

Joseph A. Nyberg of the Hyde Park High School, Chicago, has just published with the American Book Company, *A Second Course in Algebra.*

E. C. Rushmer of the Central High School, Binghamton, New York, and C. J. Dence of the Central High School, Syracuse, New York, have published with the American Book Company, *A Second Course in Algebra.*

Professor William L. Hart of the University of Minnesota has published with D. C. Heath and Company a new text, *College Algebra.*

Mr. Julius J. H. Hayn, head of the Mathematics Department of the Masten Park High School, Buffalo, New York, has just published a text book in geometry entitled *A Geometry Reader.* The book is published by the Bruce Publishing Company of Milwaukee.

Professor Ralph F. Newcomb of the State Teachers College at Ada, Oklahoma, has written a new text, *Modern Methods of Teaching Arithmetic,* which has recently been published by Houghton-Mifflin Company in their Riverside Text Book Series. /S

Joseph F. Gonnelly, District Superintendent in Charge of Junior High Schools in Chicago and Miss L. Grace Huff of the Lane Technical High School, Chicago, have published with Charles E. Merrill Company a series of books in Junior High School Mathematics.

The regular spring meeting of the Philadelphia Section of Teachers of Mathematics of Middle States and Maryland was held Saturday, March 13, 1926, at 9:30 in Central High School for Boys. The President, Mr. Brecht, presided.

Dr. Samuel G. Barton, Acting Director of Flower Observatory of University of Pennsylvania, was the first speaker. Dr. Barton

gave an interesting address on "The Determination of Celestial Distances" which was illustrated with slides.

The second speaker was Mr. Donald E. MacCormick, Head of the Department of Mathematics of the William Penn Charter School. Mr. MacCormick spoke on "The How and Why of the College Entrance Board."

The program was followed by a short business meeting. The secretary's minutes were read and accepted. The treasurer's report, showing a balance of $42.24, was accepted. The nominating committee consisting of Dr. J. T. Rorer, Dr. H. E. Shoemaker and Miss Mildred Barlow, reported as follows:

For President—Mr. Samuel K. Brecht of Central High School for Boys;

Vice-President—J. Albert Blackburn of Friend's Central School.

Secretary-Treasurer—Miss Julia Bligh of South Philadelphia High School for Girls;

Member of Executive Committee—Miss Anna Sensenig of Philadelphia Girls High School.

It was moved, seconded and carried that the report of the nominating committee be accepted and officers declared elected.

Professor E. R. Hedrick of the University of Southern California, is scheduled to offer two courses in Columbia University during the 1926 summer session. These courses are entitled: *Fundamental Concepts of Modern Mathematics* and *Theory of Functions of a Complex Variable.*

The stenographic firm of Rexford L. Holmes, Inc., 1414 Crittenden St., N. W., Washington, D. C., will supply a verbatim copy of the proceedings of the recent annual meeting of the National Council of Teachers of Mathematics, including a copy of the Yearbook, for $10.25 per copy, or $25.50 for three copies sent to the same address. The report includes everything said and read from platform and floor, and is not unduly repetitive of the papers set forth in the Yearbook, but is rather explanatory thereof and supplemental thereto. All colleges and universities, junior high schools, senior high schools and all teachers of mathematics, will desire to have a copy of this invaluable report of proceedings.

MEMBERS OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

NEW JERSEY

(Continued from March Number)

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Senior High School, Hamilton Avenue, Trenton
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Elmer Schuyler, Bay Ridge High School, 67th Street and 4th Avenue, Brookyn
Maud Randles, 4517 Sixth Avenue, Brooklyn
Fannie I. Brown, 408 Eighth Avenue, Brooklyn
Sister Mary Thecla S. S. J., St. Agnes Seminary, 287 Union Street, Brooklyn
Clara L. Crampton, 95 Gates Avenue, Brooklyn
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Boys' High School, Marcy and Putnam Avenues, Brooklyn
Stephen Emery, Erasmus Hall High School, Flatbush Ave., Brooklyn
Arthur Haas, Thomas Jefferson High School, Brooklyn
Jennie Ehlers, 99 Autumn Avenue, Brooklyn
Mary M. Oliver, 629 St. Marks Avenue, Brooklyn
George F. Wilder, Erasmus H. S., Church and Flatbush Avenues, Brooklyn
Augusta Homnick, 1421-53rd Street, Brooklyn
C. P. Scoboria, Polytechnic Preparatory Country Day School, Dyker Heights, Brooklyn
A. C. Ledner, 196 Clinton Avenue, Apt. A-53, Brooklyn
Miss Carrigan, 141 McDonough Street, Brooklyn
Margaret Allen, Parker Collegiate Institute, 170 Joralemon Street, Brooklyn
Miss G. Caldwell, Parker Collegiate Institute, 170 Joralemon Street, Brooklyn
Helen Crossman, Parker Collegiate Institute, 170 Joralemon Street, Brooklyn
L. Leland Locke, 950 St. Johns Place, Brooklyn
Sister M. Pancratia, Mount Mercy Academy, 1475 Abbott Road, Buffalo
Harriet Bull, Masten Park High School, Buffalo
Mary Crofts, 124 Woodbridge Avenue, Buffalo
McKinley Vocational School, 1500 Elmwood Avenue, Buffalo
Miss Hallie S. Poole, 674 Richmond Avenue, Buffalo
Albert Wanemacher, 23 Coe Place, Buffalo
The Nichols School of Buffalo, Amherst and Colvin Streets, Buffalo
Clara I. Roeper, 26 Huntington Avenue, Buffalo
Maude G. Lewis, 3059 Delaware Avenue, Buffalo
Grosvenor Library, Buffalo
Mable McCurdy, Technical High School, Buffalo
Margaret Wilkins, 74 Claremont Avenue, Buffalo
Katharine L. Barcalo, 779 Auburn Avenue, Buffalo
Mary A. Carter, 192 Hughes Avenue, Buffalo
Bertha C. Fiore, 73 Clinton Street, Buffalo
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Lulu J. Comstock, 40 Fort Hill Avenue, Canandaigua
Library, St. Lawrence University, Canton
Abbie M. Fellows, Drew Seminary, Carmel
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Mary R. Smith, Chazy
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L. Wida Zimmer, Box No. 357, Constableville
Helen N. Hale, 50 Chestnut Street, Cooperstown
Miss Esther B. Barnes, 24 Freileigh Place, Coxsackie
Miss L. C. Montgomery, Croton-on-Hudson
Irving M. Chriswell, R. F. D. No. 12, Darien Center

Mabel E. Reed, 230 Park Place, East Aurora
Alice E. Sherman, 126 Whaley Avenue, East Aurora
Helen M. Hubbard, 701 College Avenue, Elmira
Martha E. Martin, 20 Gay Street, Elmhurst, Long Island
E. Grant Spicer, Boys' School, Lake Pacid Club, Essex County
Ernie M. Boardman, Falconer
R. W. Williams, 86 Elm Avenue, Flushing
Prof. William Pitt Durfee, 639 Main Street, Geneva
Sara A. Keiner, 27 Kingsboro Avenue, Gloversville
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Edith M. Phillips, 18 Allen Street, Jamestown
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Mary M. Bowe, 189 Lakeview Avenue, Rockfield Center, Long Isand
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Christina E. Carey, 1 Lawrence Avenue, Potsdam

(To be continued)